

Constraining CP violating phases of the MSSM*

I. Hinchliffe and N. Kersting

Lawrence Berkeley National Laboratory, Berkeley, CA

Abstract

Possible CP violation in supersymmetric (SUSY) extensions of the Standard Model (SM) is discussed. The consequences of CP violating phases in the gaugino masses, trilinear soft supersymmetry-breaking terms and the μ parameter are explored. Utilizing the constraints on these parameters from electron and neutron electric dipole moments, possible CP violating effects in B -physics are shown. A set of measurements from the B -system which would over-constrain the above CP violating phases is illustrated.

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1 Introduction

The peculiar flavor structure of the Standard Model (SM) currently has no generally-accepted explanation. Included in this structure is the prediction that fundamental physics is not invariant under the operations of parity(P), charge(C), or their combination, CP. More precisely, it is the distribution of complex numbers in the SM Lagrangian which lead to CP violation at an extremely small [†] but observable level [1]. Currently experiments [2, 3, 4, 5, 6, 7, 8] are taking data or under construction that will very precisely measure the Cabbibo-Maskawa-Kobayashi (CKM) [9] matrix elements believed responsible for CP violation in the SM. Any observed discrepancy with the SM prediction indicates that the SM is incomplete in the flavor sector and new physics must appear in the fundamental theory. An excellent candidate for new physics is Supersymmetry (SUSY) [10]. As SUSY introduces many more potentially large sources of CP violation in the mass matrices and various field couplings, its inclusion requires that there be some suitable relationships among the parameters to reproduce the agreement of the existing levels of CP violation in K -decay and electric dipole moment (EDM) data with the SM predictions [11]. The understanding of what CP violating parameters are allowed in SUSY models therefore provides a constraint on such models.

The simplest SUSY models can not satisfy the experimental EDM bounds without either setting all SUSY CP violating phases to zero or raising SUSY particle masses above 1 TeV [12, 13, 14]. Recent works have noted, however, that the supergravity-broken MSSM with $O(1)$ phases for the gaugino masses M_i , triscalar coupling A , and Higgs coupling parameter μ , and no flavor-mixing beyond the standard CKM matrix can be consistent with existing limits on the electric dipole moments (EDMs) of the electron and neutron [15, 16]. Given this, we may now investigate how the phases in this model will contribute to other CP violating observables and how new measurements can constrain the values of these phases. We choose to investigate several CP violating observables of the B -meson system, as B -processes occupy both a favorable theoretical and experimental position. On the theoretical side, uncertainties from non-perturbative QCD are low due to the large mass of the b -quark ($\approx 4 GeV$) [17] relative to the energy scale $\Lambda \approx 200 MeV$ [18] characteristic of strong interactions and perturbation theory in α_s is reliable. Furthermore, many SM CP violating asymmetries in the B -system are small due to the suppression of CKM matrix elements involving the third generation; new CP violating physics may be easily detectable. Experimentally, many dedicated facilities are already or will soon be generating large amounts of high-precision B -physics data [19]. From our analysis we will see that these experiments will be able to either precisely determine the above set of five phases or reject this particular model of CP violation altogether.

We first briefly discuss the features of the MSSM in Section 2 and review the EDM constraints in Section 3. In Section 4 we begin discussion of how the above phases enter observables in various sectors of B -meson physics through CP violation in pure mixing, mixing and decay, and pure decay effects in the processes $b \rightarrow s \gamma$, $B_s^0 \rightarrow \phi \phi$, $J/\psi \phi$, and $B^- \rightarrow \phi K^-$. Finally, we collect results and compare with the expected experimental sensitivities in Section 5. Throughout this discussion we reserve most of the more complicated formulae and analyses for the Appendix, to which we will refer the reader at the appropriate points.

[†]Jarlskog [1] has expressed this as follows: considering the interaction-basis up- and down-quark mass-matrices m , m' , one can form a rephase-invariant measure of CP violation equal to $a \equiv 3\sqrt{6}Det(C)/(Tr(C^2))^{3/2}$ where $iC \equiv [mm^\dagger, m'm'^\dagger]$. Then $-1 \leq a \leq +1$ and CP violation $\Leftrightarrow a \neq 0$. In the SM one finds $a \approx 10^{-7}$

2 The Model

The setting for our calculations is the MSSM superpotential

$$\mathbf{W} = \mathbf{y}_u \bar{u} Q H_2 + \mathbf{y}_d \bar{d} Q H_1 + \mathbf{y}_e \bar{e} L H_1 + \mu H_1 H_2 \quad (1)$$

with the chiral matter superfields for quarks and leptons \bar{u} , \bar{d} , Q , \bar{e} , L , and Higgs $H_{1,2}$ transforming under $SU(3)_C \times SU(2)_L \times U(1)_Y$ as:

$$\begin{aligned} \bar{u} &\equiv (\bar{3}, 1, -\frac{2}{3}) & \bar{d} &\equiv (\bar{3}, 1, \frac{1}{3}) & Q &\equiv (3, 2, \frac{1}{6}) \\ \bar{e} &\equiv (1, 1, 1) & L &\equiv (1, 2, -\frac{1}{2}) \\ H_1 &\equiv (1, 2, -\frac{1}{2}) & H_2 &\equiv (1, 2, \frac{1}{2}) \end{aligned} \quad (2)$$

The 3×3 Yukawa matrices $\mathbf{y}_{u,d,e}$ couple the three generations of quarks and leptons, and the μ parameter couples the two Higgs. In this discussion all flavor and gauge indices are implicit. The superpotential (1) and general considerations of gauge invariance give the minimally supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y$ Lagrangian

$$\mathcal{L}_{SUSY} = \mathcal{L}_{kinetic} - \sum_i \frac{dW}{d\phi^i} \frac{dW^*}{d\phi^i} - \frac{1}{2} \sum_a g_a^2 (\phi^* T^a \phi)^2 \quad (3)$$

where the kinetic terms have the canonical forms

$$\mathcal{L}_{kinetic} = -D^\mu \phi^{\dagger i} D_\mu \phi_i - i\psi^{\dagger i} \bar{\sigma}^\mu D_\mu \psi_i - \frac{1}{4} F^{\mu\nu a} F_{\mu\nu a} \quad (4)$$

with the indices i and a running over the scalar fields and gauge group representations, respectively. To this Lagrangian we add a soft supersymmetry-breaking ($SUSY$) piece

$$\begin{aligned} \mathcal{L}_{SUSY} = & (M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B}) - (\mathbf{a}_u \bar{u} Q H_2 + \mathbf{a}_d \bar{d} Q H_1 + \mathbf{a}_e \bar{e} L H_1)_{scalar} \\ & - (\sum_{X_i=Q, \bar{u}, \bar{d}, \bar{e}} \mathbf{m}_i^2 X_i X_i^\dagger - \sum_{i=1,2} m_{H_i}^2 H_i^* H_i - b\mu H_1 H_2)_{scalar} + h.c. \end{aligned} \quad (5)$$

where $(\tilde{g}, \tilde{W}, \tilde{B})$ are the fermionic partners of the gauge bosons of $SU(3)_C, SU(2)_L, U(1)_Y$, respectively, and where *scalar* implies that only the scalar component of each superfield is used. SUSY-breaking of this type is characteristic of supergravity [20, 21].

An additional simplification is to take the matrices \mathbf{m}_i^2 and $\mathbf{a}_{u,d,e}$ in (5) along with the Yukawa matrices $\mathbf{y}_{u,d,e}$ in (1) to be simultaneously diagonal in the mass basis after electroweak symmetry breaking. This implies

$$\begin{aligned} \mathbf{m}_i^2 &\equiv m_0^2 \mathbf{1} \\ \mathbf{a}_i &\equiv A \mathbf{y}_i \quad (i = u, d, e) \end{aligned} \quad (6)$$

Thus the only mixing between families is the usual CKM matrix since quark and squark mass matrices are diagonalized by the same rotation.

The full Lagrangian $\mathcal{L}_{SUSY} + \mathcal{L}_{SUSY}$ has six arbitrary phases imbedded in the parameters $M_i (i = 1, 2, 3)$, μ , b , and A . The phase of b can immediately be defined to be zero by appropriate field redefinitions of $H_{1,2}$. A mechanism to set another phase to zero utilizes the $U(1)_R$ -symmetry of the Lagrangian which is broken by the supersymmetry-breaking terms, in particular the gaugino masses in (5). We may perform a $U(1)_R$ rotation on the gaugino fields to remove one of the phases M_i . For consistency with [15] we choose M_2 real ($\phi_2 \equiv 0$). Note that this $U(1)_R$ transformation affects neither the phase of A , since having the Yukawa matrices \mathbf{y}_i in (1) be real fixes the phases of the same fields that couple to A , nor the phase of μ , as having chosen $\phi_b \equiv 0$ fixes the phases of $H_{1,2}$. Therefore the final set of physical phases we study is $\{\phi_{1,3}, \phi_A, \phi_\mu\}$.

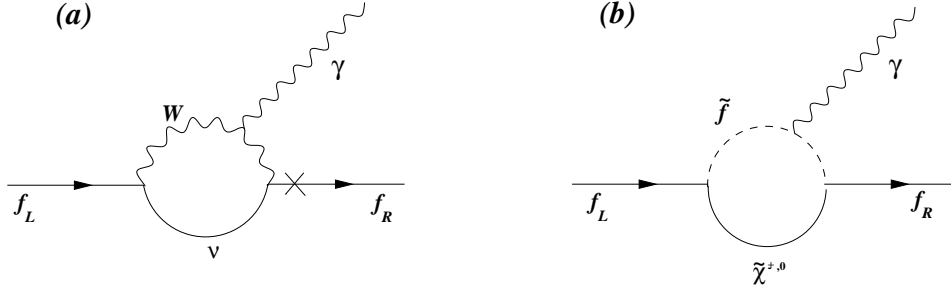


Figure 1: 1-Loop Contributions to the EDM in (a) the SM; the graph vanishes since the complex phases at the vertices cancel (b) in the MSSM, where the requisite helicity-flip may be placed on either the chargino ($\tilde{\chi}^\pm$), neutralino ($\tilde{\chi}^0$), or scalar (\tilde{f}) propagators to introduce $O(1)$ phases in the amplitude.

3 Electric Dipole Moment Constraints

The electric dipole moment (EDM) of an elementary fermion, a manifestly CP violating quantity, is the coefficient d_f of the effective operator

$$O_{edm} = -(i/2)\bar{f}\gamma_5\sigma_{\mu\nu}fF^{\mu\nu}$$

where $F^{\mu\nu}$ is the electromagnetic field-strength tensor. The SM prediction of this coefficient vanishes at one loop (see Figure 1a) since the CKM phases from the two vertices cancel each other. For the electron, d_e even vanishes at two loops and the three-loop prediction is miniscule, of order $10^{-50}ecm$ [22]. For the neutron EDM, gluon interactions can give rise to a two-loop contribution to d_n , but the result is still tiny at $d_n \leq 10^{-33}ecm$ [23]. The above predictions no longer hold if the QCD Lagrangian contains the CP violating ‘ θ -term’ $\theta \frac{g_{QCD}^2}{32\pi^2} G^{\mu\nu} \tilde{G}_{\mu\nu}$, a potentially significant source of CP violation, but then the theory is consistent with the EDM limits only if $\theta < 10^{-9}$ [24]; such a fine-tuning is unnatural and henceforth we assume that $\theta = 0$.

Although the amplitude of a typical supersymmetric contribution to the dipole moment (see Figure 1b) suffers relative to the SM contribution by a reduction factor of at least $(m_W/\tilde{m})^2$ from SUSY particles of mass \tilde{m} propagating in the loop, the imaginary piece of the SUSY amplitude could very well dominate over that from the SM. As previous calculations have shown [12, 13, 14], the diagrams in Figure 1b lead to EDMs in violation of the current limits of $d_e < 4.3 \times 10^{-27}ecm$ [25] and $d_n < 6.3 \times 10^{-25}ecm$ [26] unless SUSY particles are heavier than $O(1)TeV$ or the CP violating phases are less than $O(10^{-2})$. However, small phases in fact are not inevitable, for recently it has been pointed out [15, 16] that a cancellation can occur among the various SUSY diagrams, allowing $O(1)$ CP violating phases to be consistent with the current EDM experimental limits.

The set of phases $\{\phi_{1,3}, \phi_A, \phi_\mu\}$ enter in any diagram which involves mixing between the following fields:

- Charginos ($\tilde{\chi}^\pm$): ϕ_μ lies in the matrix \mathbf{M}_{χ^\pm} which mixes the set $(\tilde{W}^+, \tilde{H}_2^+, \tilde{W}^-, \tilde{H}_1^-)$
- Neutralinos ($\tilde{\chi}^0$): both ϕ_μ and ϕ_1 appear in the matrix \mathbf{M}_{χ^0} which mixes the set $(\tilde{W}^0, \tilde{B}, \tilde{H}_1^0, \tilde{H}_2^0)$
- Scalars: terms in the Lagrangian such as $\mu^* \mathbf{y}_u \tilde{u} \tilde{u} H_1^{0*}$ arising from the second term of (3), and $SUSY$ -terms in (5) introduce ϕ_μ and ϕ_A , respectively, into the scalar mass insertions. The effect is particularly significant in \tilde{t} -mixing where the Yukawa matrices are large.

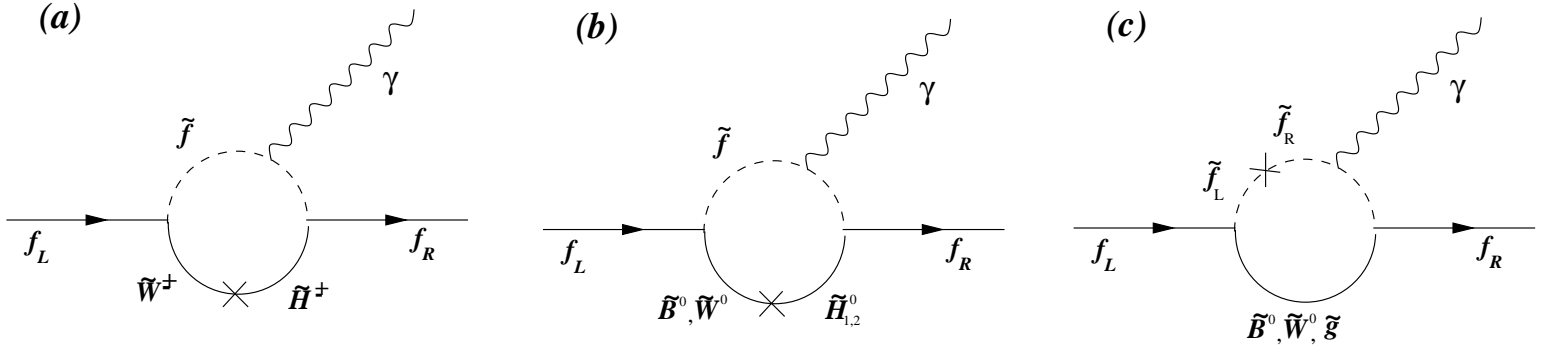


Figure 2: Leading SUSY Contributions to the EDM: (a) charged wino (\tilde{W}^\pm) and higgsino (\tilde{H}^\pm) mixing provide the largest part of d_f which (b) neutral wino (\tilde{W}^0), bino (\tilde{B}), and higgsino (\tilde{g}^0) mixing partially cancels. (c) exchange of a \tilde{W}^0 , \tilde{B} , or gluino \tilde{g}^0 with mixing between the scalar superpartners ($\tilde{f}_{L,R}$) of the corresponding fermions $f_{L,R}$ could almost completely cancel against the other two diagrams. Note that the above processes actually occur via the *mass* eigenstates $\tilde{\chi}^\pm$ and $\tilde{\chi}^0$.

The SUSY diagrams which appear in Figure 1(b) fall into three classes, shown in Figure 2. The charged wino-higgsino mixing diagram Figure 2(a) provides the dominant contribution to d_f , with the phase of μ entering the amplitude as expected. The neutralino-mixing diagram Figure 2(b) is numerically smaller than its charged counterpart, but it is of opposite sign and has the same dependence on ϕ_μ ; therefore 2(a) and 2(b) partially cancel. The final type of diagram shown in Figure 2(c) has a more complicated phase dependence[†] which, in certain regions of parameter space that are not fine-tuned, leads to a destructive interference with the other two diagrams consistent with current experimental bounds on *both* d_n and d_e [15].

If $O(1)$ phases are then permissible, it is important to know whether or not other experimental observables can overconstrain these phases. We next show that the B system alone provides enough observables to strongly constrain these phases.

4 B Physics Constraints

There are many reasons to consider the B -system in particular for measurements of CP violation beyond the SM, a few of which are:

- SM contributions to the relevant observables are down by factors of small CKM matrix elements, so generic non-SM CP violating physics should give a very clear signal.
- uncertainties arising from strong interactions are small using Heavy Quark Theory [27]
- large amounts of data will be available in the near future

The CP violation in question can arise in any of three ways: through $B^0 - \bar{B}^0$ mixing, B -decay, or through their combination [2, 27].

[†]we briefly outline this dependence in the Appendix

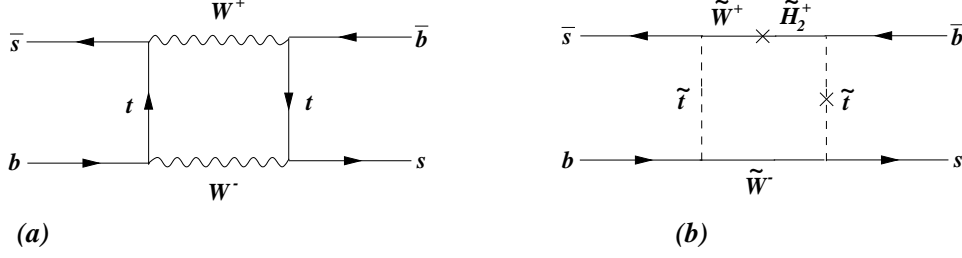


Figure 3: Leading box diagrams in (a) the SM, and (b) the MSSM

4.1 CP violation in $B^0 - \bar{B}^0$ mixing

At any given instant of time, the propagating meson states are linear combinations $|B_{L,H}\rangle \equiv p|B^0\rangle \pm q|\bar{B}^0\rangle$ which evolve according to the 2×2 Hamiltonian with dispersive and absorptive pieces \mathbf{M} and $\mathbf{\Gamma}$, viz

$$i \frac{d}{dt} \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = \begin{pmatrix} M_{11} + \frac{i}{2}\Gamma_{11} & M_{12} + \frac{i}{2}\Gamma_{12} \\ M_{12}^* + \frac{i}{2}\Gamma_{12}^* & M_{11} + \frac{i}{2}\Gamma_{11} \end{pmatrix} \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} \quad (7)$$

If we denote the time evolution of the states as

$$|B_{L,H}(t)\rangle = |B_{L,H}(0)\rangle e^{-iM_{L,H}t} e^{-\frac{1}{2}\Gamma_{L,H}t} \quad (8)$$

then solving (7) it follows [2] that

$$CP \text{ violation in mixing} \iff \text{Im}(\Delta M) \neq 0 \quad (\Delta M \equiv M_L - M_H) \quad (9)$$

Making use of the fact that

$$\Delta\Gamma \ll \Delta M \quad (\Delta\Gamma \equiv \Gamma_L - \Gamma_H),$$

which holds for both B_d^0 and B_s^0 [2, 28], we obtain the simplifications

$$\Delta M \approx 2|M_{12}| \quad \frac{q}{p} \approx \frac{-|M_{12}|}{M_{12}} \quad (10)$$

The degree of CP violation in mixing is contained in the ratio $\frac{q}{p}$, which from (10) is directly proportional to the phase factor in the amplitude of the $\Delta S = 2$ box diagram (see Figure 3). In the SM, the box graph with internal top quarks in Figure 3(a) dominates $\text{Im}(\Delta M)$; correspondingly, the strength of SM CP violating mixing is CKM-suppressed by a factor $\text{Arg}(V_{tb}V_{ts}^*)^2 \approx \eta\lambda^2$ which in the Wolfenstein approximation [29] is ≈ 0.01 . Any $\Delta S = 2$ contribution from new physics therefore has a generically large effect if it carries any phases with it at all. The leading supersymmetric chargino graph[§] in Figure 3(b) provides the largest MSSM contribution to $\text{Im}(\Delta M)$. We have computed this box (see Appendix) and find it to dominate significantly over the SM contribution. For example, at a typical point in SUSY parameter space, with $A \approx \mu$, $\tan\beta = 5$, and sparticle masses \tilde{m} on the order of $O(2) \times M_W$, we obtain

$$\text{Im}\left(\frac{q}{p}\right) \approx 0.1 \sin\phi_\mu \cos\phi_A \quad (11)$$

[§]We neglect gluino boxes in the approximation that SUSY introduces negligible flavor mixing in the down-sector; boxes with additional Higgsinos are likewise suppressed by small Yukawa matrices. Further, we assume that the lightest stop dominates the loops since it is usually the lightest squark.

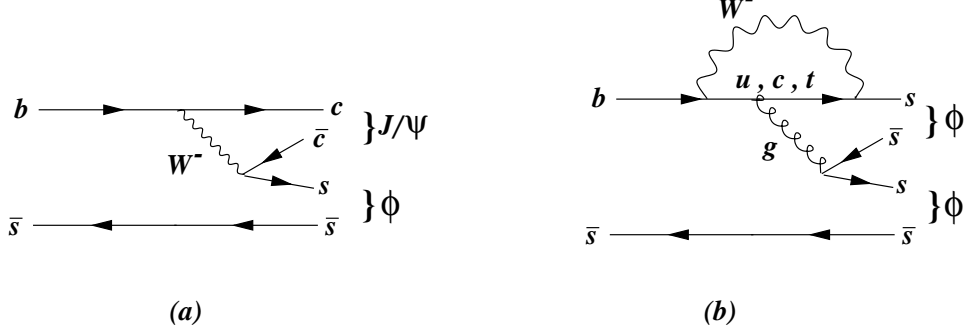


Figure 4: Sizable CP violating asymmetries arise from the processes (a) $\overline{B}_s^0 \rightarrow J/\psi \phi$, and (b) $\overline{B}_s^0 \rightarrow \phi\phi$. Only in the first case is hadronic uncertainty absent.

which is an order of magnitude larger than the SM expectation and is directly sensitive to CP violating SUSY phases. However, it is difficult to directly measure $Im\left(\frac{q}{p}\right)$ through mixing effects alone; both time-dependent and time-integrated mixing effects are usually governed by ΔM which is still mostly real and SM-driven. We must therefore turn to other types of CP violation to constrain the MSSM phases.

4.2 CP violation from mixing combined with decay

When a particular final state f_{CP} with definite CP quantum numbers is accessible to the decays of a B^0 and \overline{B}^0 with amplitudes $A_{f_{CP}}$ and $\overline{A}_{f_{CP}}$, respectively, the asymmetry

$$a_{f_{CP}} \equiv \frac{\Gamma(\overline{B}^0 \rightarrow f_{CP}) - \Gamma(B^0 \rightarrow f_{CP})}{\Gamma(\overline{B}^0 \rightarrow f_{CP}) + \Gamma(B^0 \rightarrow f_{CP})}$$

in the limit of (10) becomes

$$a_{f_{CP}} \approx \sin(\Delta Mt) Im\left(\frac{q}{p} \overline{\rho}_{f_{CP}}\right) \quad (12)$$

where

$$\overline{\rho}_{f_{CP}} \equiv \frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}} \quad A_{f_{CP}}(B^0 \rightarrow f_{CP}) \equiv \sum_j A_j e^{i(\delta_j + \phi_j)}$$

in general contains a dependence on both the weak phases ϕ_j and the strong interaction phases δ_j from each diagram contributing an amplitude A_j . In Figure 4, for example, the decay of a \overline{B}_s^0 to the final states $J/\psi \phi$ and $\phi\phi$ may either proceed directly ($\overline{B}_s^0 \rightarrow f_{CP}$) or by oscillating first ($\overline{B}_s^0 \rightarrow B_s^0 \rightarrow \overline{B}_s^0 \rightarrow f_{CP}$). In the SM the asymmetries in both of these decays are tiny primarily due to the small $\Delta S = 2$ mixing effects (see discussion following (10) which effectively give the upper bound

$$a_{\phi\phi, J/\psi\phi}(SM) \leq \eta\lambda^2 \approx 0.01 \quad (13)$$

If SUSY exists then both of these asymmetries can be an order of magnitude larger as is evident from (11) and (12). Furthermore, the value of the quantity $\overline{\rho}_{f_{CP}}$ will differ from its value in the heavy quark expansion (HQE) by powers of Λ_{QCD}/m_b only [30, 31, 32], so we set the strong phases at $\delta_j = \pi$ with an uncertainty $(\Delta\delta)/\delta < 10\%$ [33, 34]. In the case of the decay to $\phi\phi$ Figure 4(b), for example, $\overline{\rho}_{f_{CP}} \approx 1$, whereas for the decay to $J/\psi \phi$ through the dominant tree-graph Figure 4(a) carries no strong-phase dependence at all. From (11) and (12) it follows as a

prediction of our model that

$$a_{J/\psi \phi, \phi \phi} \approx 0.1 \sin \phi_\mu \cos \phi_A \quad (14)$$

We now have two experimental B -physics signals of new CP violating phases which constrain a combination of ϕ_μ and ϕ_A independent from those which arise in the EDM bounds.

4.3 CP violation in decay

The charged B -mesons' decays can also serve to measure our set of phases. The asymmetry in the decay to a final state f is

$$a_f \equiv \frac{1 - |\bar{A}/A|^2}{1 + |\bar{A}/A|^2}$$

$$A \equiv \text{Amp}(B^+ \rightarrow f) = \sum_j A_j e^{i(\delta_j + \phi_j)}$$

$$\bar{A} \equiv \text{Amp}(B^- \rightarrow \bar{f}) = \sum_j A_j e^{i(\delta_j - \phi_j)}$$

with δ_j and ϕ_j being the strong and weak phases, respectively for the diagram with modulus A_j . Rewritten in the form

$$a_f = \frac{\sum_{i,j} A_i A_j \sin(\phi_i - \phi_j) \sin(\delta_i - \delta_j)}{\sum_{i,j} A_i A_j \cos(\phi_i - \phi_j) \cos(\delta_i - \delta_j)} \quad (15)$$

it is clear that a non-zero asymmetry requires that at least two diagrams contribute with different strong *and* weak phases.

4.3.1 $b \rightarrow s \gamma$

One of the interesting features of this mode is that any physical model that introduces new CP violating phases can result in an asymmetry far greater than that which the SM predicts; yet it must not disturb the branching ratio(BR) for $b \rightarrow s \gamma$ which CLEO has measured [35]:

$$BR(b \rightarrow s \gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4} \quad (16)$$

In Figure 5 we show the dominant diagrams; as in the case of the SUSY contributions to the EDMs studied above, the most important non-SM diagrams for $b \rightarrow s \gamma$ involve the chargino loop in Figure 5(b) [37]. Since the observed value of the branching ratio (16) agrees very well with the SM prediction, new decay channels are strongly constrained; accordingly, we follow the analysis of [36] in carefully accounting for the higher-order graphs in Figure 5(c,d,e).

The effective operators involved are

$$\begin{aligned} O_2 &\equiv \bar{s}_L \gamma_\mu q_L \bar{q}_L \gamma^\mu b_L \\ O_7 &\equiv \frac{em_b}{4\pi^2} \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R \\ O_8 &\equiv \frac{gs m_b}{4\pi^2} \bar{s}_L \sigma_{\mu\nu} G^{\mu\nu} b_R \end{aligned} \quad (17)$$

We leave the evaluation of these operators and all related calculations for the Appendix.

In using these diagrams to compute observables it is important to take into account that the photon involved in the decay $b \rightarrow s \gamma$ is monochromatic but the photon in the observable $B \rightarrow X_s \gamma$ has a variable energy. In addition to depending on the final state X_s , the photon energy is also a function of how the b -quark is bound inside the B -meson; if the recoil energy of the b -quark is small,

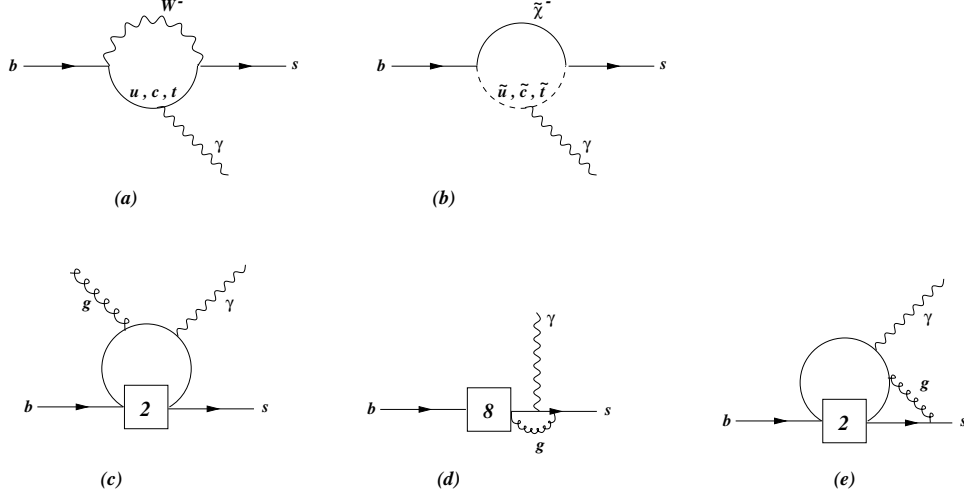


Figure 5: Leading contributions to $b \rightarrow s \gamma$ in (a) SM (b) MSSM. Graphs (c), (d), and (e) with the effective vertices $O_{2,8}$ play a significant role as well (mass insertions understood)

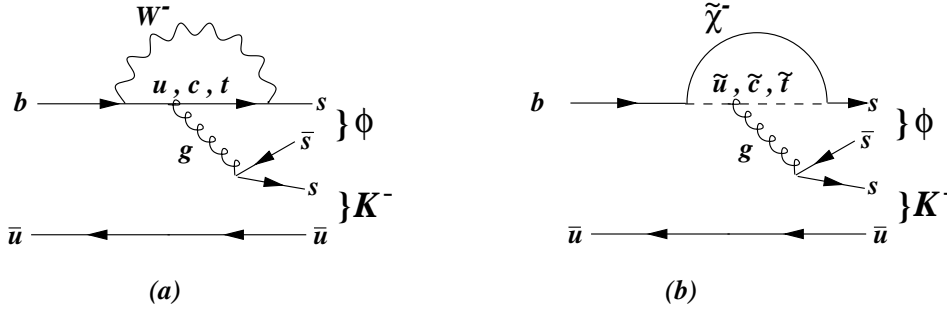


Figure 6: $B_s^- \rightarrow K^- \phi$ in (a) the SM, where the CP violating asymmetry is dominated by the CKM-suppressed u -quark diagram and (b) the MSSM, where the asymmetry is dominated by the relatively CKM-unsuppressed \tilde{c} and \tilde{t} graphs. Here mass insertions are understood.

non-perturbative effects arise for which no reliable models currently exist. These considerations lead us to perform all calculations using a variable outgoing photon energy, E_γ , which is bounded from below: $E_\gamma > (1 - \xi)E_{max}$, where ξ is between 0 and 1 and E_{max} is a model-dependent quantity. The actual dependence on ξ and E_{max} in the computed asymmetry $a_{b \rightarrow s \gamma}$ turns out to be negligible, as we demonstrate in the Appendix. Here we present the result

$$a_{b \rightarrow s \gamma} \approx 0.01 \sin(\phi_\mu)$$

where as in the case $B_s^0 \rightarrow \phi \phi$ the strong phase dependence is included in the coefficient and imparts a 10% uncertainty to this prediction. The magnitude of the asymmetry is admittedly not very large, however it is at least twice the SM prediction [38].

4.3.2 $B^- \rightarrow K^- \phi$

One particularly striking signature of the presence of the SUSY phases is in the decay $B^- \rightarrow K^- \phi$. Here the flavor structure $b \rightarrow s \bar{s} s$ forbids a SM tree graph, so the leading SM graph is a penguin (defined ‘P’), as shown in Figure 6(a). The leading SUSY contribution in Figure 6(b) is also

a type of penguin (hereafter designated a superpenguin, ‘SP’), suppressed however by a factor $(m_W/\tilde{m})^2$ relative to the SM due to a squark propagating in the loop instead of a W -boson. So far the situation parallels the decay $B^0 \rightarrow \phi \phi$ above, but the major difference is that here the CP violation is necessarily *direct*. Referring back to (15), with $\{i, j\}$ running over $\{u, c, t, \tilde{u}, \tilde{c}, \tilde{t}\}$, we see that the asymmetry receives contributions from 36 interference terms. However the imaginary parts of SP-SP interference terms are zero since the squarks in the loops of SP graphs are heavier than the b -quark and do not give rise to absorptive phases in the amplitude (*i.e.* the δ ’s are zero in (15)). This leaves SP-P and P-P terms. The latter, being purely SM terms, must always involve at least one u -quark to get a nonzero weak phase ($V_{tb}V_{ts}^* = -V_{cb}V_{cs}^* = -A\lambda^2$ is real), whereas the former need not involve u -quarks since the weak phase difference necessary for a non-zero asymmetry can come from a SUSY coupling in \tilde{c} or \tilde{t} graphs. Therefore the dominant SP-P terms will (in the notation of 15) have $i \in \{\tilde{c}, \tilde{t}\}$ and $j \in \{c, t\}$, leading over the P-P terms by a factor $(V_{tb}V_{ts}^*)/Im(V_{bu}V_{us}^*) \approx 1/\eta\lambda^2 \approx 100$. Therefore the P-P terms are negligible and the asymmetry is fundamentally due to the SUSY-SM interference. Assuming as before that the lightest \tilde{t} dominates the SP, the numerator of (15) only contains the term due to \tilde{t} - c interference:

$$a_{B^- \rightarrow K^- \phi} \approx \frac{A_c A_{\tilde{t}} \sin(\phi_c - \phi_{\tilde{t}}) \sin(\delta_c)}{A_c^2} \quad (18)$$

We now employ the one-loop perturbative calculation of the strong phase δ_c from the c -quark loop [34, 33], and since $\phi_c \approx \eta\lambda^2$ is negligible and $\phi_{\tilde{t}}$ is essentially the same phase we calculated for $b \rightarrow s \gamma$ [¶] we obtain

$$a_{B^- \rightarrow K^- \phi} \approx \left(\frac{M_W}{\tilde{m}} \right)^2 \sin(\phi_\mu) \quad (19)$$

Here we see that the absence of neutral flavor changing currents, the hierarchy of the CKM matrix, and the pattern of CP quantum numbers all conspire in this case to give an asymmetry which is essentially zero in the SM yet for SUSY with typical sparticle masses can be as large as 30%!

5 Discussion

We have seen that the B -system observables above provide many constraints on the phases $\{\phi_A, \phi_\mu\}$ in the model. We may classify the experiments by the size of the event samples they are expected to provide:

High Luminosity(HL): For example experiments at the LHC p-p collider, producing a sample of order 10^{10} $B^0 - \overline{B}^0$ pairs per year [8]

Low Luminosity(LC): Experiments run at e^+/e^- machines such as CESR(CLEO III), KEKB(Belle), and PEP-II(BaBar) as well as at hadronic machines such as Fermilab(CDF, DØ) and DESY(HERA-B: actually e^+p) produce similar samples of $B^0 - \overline{B}^0$ pairs, of order 10^7 per year [2, 3, 4, 5, 6]

Any experiment which detects N $B^0 - \overline{B}^0$ pairs can resolve at the 1σ -level an asymmetry a_p for a process p if

$$a_p > \frac{1}{\sqrt{BR(p)N}}$$

[¶]although the helicity structure of the ingoing b -quark and outgoing s -quark is L-R for $b \rightarrow s \gamma$, the L-L and R-R helicity structures of $B_s^- \rightarrow K^- \phi$ give negligible contributions to the *phase* of the diagram

Table 1: Predicted Asymmetries in the B -system and Experimental Error. a_p is listed for both *high luminosity*(H \mathcal{L}) and *low luminosity*(L \mathcal{L}) experiments (see text for explanation) in one year of running. The strong phase uncertainty factors $\delta_i \approx 1 \pm 0.1$. For the explicit form of the function $f(\phi_\mu, \phi_A)$ see the Appendix

$Process(p)$	$BR(p)$	$a_p(\text{predicted})$	$a_p(\text{H}\mathcal{L})$	$a_p(\text{L}\mathcal{L})$
$b \rightarrow s \gamma \quad (B^0 \rightarrow X_s \gamma)$	2×10^{-4}	$0.01 \sin(\phi_\mu) \delta_1$	0.001	0.02
$B_s^0 \rightarrow \phi \phi$	$4 \times 10^{-5} \times (0.50)^2$	$0.10 f(\phi_\mu, \phi_A) \delta_2$	0.003	0.09
$B_s^0 \rightarrow J/\psi \phi$	$10^{-3} \times (0.12) \times (0.50)$	$0.10 f(\phi_\mu, \phi_A) \delta_3$	0.001	0.03
$B_s^- \rightarrow K^- \phi$	$10^{-5} \times (0.50)$	$0.3 \sin(\phi_\mu) \delta_4$	0.004	0.12

Using this criterion, the results in Table 1 summarizes the various modes and their asymmetries, comparing them to the experimental precision expected on each asymmetry. The reader should note that the columns designated ‘H \mathcal{L} ’ and ‘L \mathcal{L} ’ refer to the *optimal* choice out of the corresponding set of experiments. In some cases experiments in the same set may have drastically different capabilities: for example the production of B_s^0 in e^+e^- annihilation requires running on the $\Upsilon(5s)$ resonance, not currently possible at BaBar or Belle, which run at the $\Upsilon(4s)$. Correspondingly the only observable in Table 1 available to these latter is the inclusive decay $b \rightarrow s \gamma$ in the decay of B_d -mesons. The decays of the daughter mesons necessary for detection of events is taken into account in the ‘ $BR(p)$ ’ column; for example, $BR(J/\psi \rightarrow (e^+ e^-, \mu^+ \mu^-)) \approx 12\%$ and $BR(\phi \rightarrow K^+ K^-) \approx 50\%$. More details on the individual capabilities of each experiment may be found in [19].

Although HQE usually yields results perturbatively convergent in powers of $\frac{\Lambda}{m_b}$, we allow for hadronic uncertainties at the level of 10% on all observables in Table 1 [36, 27, 34]. The parameters describing this are the δ_i ($i = 1..4$) which are in general completely independent for the observables in question. A valid test of the model requires a 10% measurement of the various asymmetries. Disagreement at higher precision could be ascribed to uncertainties in the strong dynamics.

From the table, we see that L \mathcal{L} -experiments can only contribute in the asymmetries in $B_s^- \rightarrow K^- \phi$ and $B_s^0 \rightarrow J/\psi \phi$. Combined with the H \mathcal{L} measurements of all of the decays studied, a determination of ϕ_μ and ϕ_A will be possible. Combined with the linear combination of phases which electron and neutron EDM’s constrain (see Appendix) this provides a complete determination of the set of the phases $\{\phi_{1,3}, \phi_A, \phi_\mu\}$ studied in this model of CP violation.

In summary, the possibility that the phase structure of the MSSM extends beyond the trivial one where CP violation is confined to the CKM matrix leads not only to the requirement that SUSY phases respect the present EDM bounds, but also that the range of phases consistent with these bounds agrees with the values which can be extracted from the various B -system asymmetries considered above. Collectively, measurements of these asymmetries at present and future B -physics experiments will either determine the phases or rule out this particular SUSY model.

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Appendix

1. Cancellation of the EDMs

That the contributions to the EDM in Figure 2(a,b) tend to cancel in a way dependent only on $\sin(\phi_\mu)$ is evident from the form of the chargino and neutralino mixing matrices:

$$\mathcal{L} \supset \tilde{\chi}^\pm T \mathbf{M}_{\tilde{\chi}^\pm} \tilde{\chi}^\pm + \tilde{\chi}^0 T \mathbf{M}_{\tilde{\chi}^0} \tilde{\chi}^0$$

where

$$\mathbf{M}_{\tilde{\chi}^\pm} \approx \begin{pmatrix} 0 & 0 & M_2 & 0 \\ 0 & 0 & 0 & \mu \\ M_2 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 \end{pmatrix} \quad \mathbf{M}_{\tilde{\chi}^0} \approx \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & 0 & -\mu \\ 0 & 0 & -\mu & 0 \end{pmatrix}$$

assuming $M_W \ll M_2, \mu$.

The graph in Figure 2(c) (where f is, say, an electron) receives phases from three sources:

- the $U(1)$ propagator carries a factor $e^{i\phi_1}$.
- the scalar mass insertion has a SUSY piece given from the first term of (3) which has the form $\mathbf{y}_e \mu^* \tilde{e} \tilde{e} H_2^{0*}$ which after EW-breaking becomes $\mathbf{y}_e \mu^* \tilde{e} \tilde{e} v \sin\beta$
- the scalar mass insertion also has a $SUSY$ piece (see (5)) of the form $\mathbf{A}_{\mathbf{y}_e} \tilde{e} \tilde{e} H_1^0$ which becomes $\mathbf{A}_{\mathbf{y}_e} \tilde{e} \tilde{e} v \cos\beta$ after EW-breaking.

Putting the above pieces together, the imaginary piece of the neutralino graph in Figure 2(c) carries a phase dependent factor $(|A| \sin(\phi_A + \phi_1) + |\mu| \tan\beta \sin(\phi_1 - \phi_\mu))$. Likewise, the corresponding graph for the neutron in the $SU(6)$ model^{||} where $d_n = 1/3(4d_d - d_u)$ obtains a factor similar to the electron case, with the replacement $\phi_1 \rightarrow \phi_3$; the numerical demonstration that these diagrams can nearly cancel is given in [15].

2. Calculating the $B_s^0 - \bar{B}_s^0$ Box

We follow the notation of [37] in calculating the contribution to M_{12} from the chargino box in Figure 3(b):

$$\Delta M_{\tilde{\chi}^\pm} = \frac{\alpha_w^2}{16} \sum_{h,k=1}^6 \sum_{i,j=1}^2 \frac{1}{m_{\tilde{\chi}^\pm}^2} (G_{UL}^{jkb} - H_{UR}^{jkb}) (G_{UL}^{*iks} - H_{UR}^{*iks}) (G_{UL}^{ihb} - H_{UR}^{ihb}) (G_{UL}^{*jhs} - H_{UR}^{*jhs}) G'_{ijkh}$$

where G and H are gauge and Higgs vertices and the form factor G' depends on the masses of the particles involved. We make the assumption that the lightest chargino and lightest stop dominate the loop, and that $M_W \ll M_2, \mu$. The final result is

$$\text{Im}(M_{12}) \approx \left(\frac{2M_W}{\tilde{m}} \right)^3 \left(\frac{\sin\phi_\mu (|A| \sin\beta \cos\beta \cos\phi_A - |\mu| \cos^2\beta y_t \cos\phi_\mu)}{\sqrt{|A|^2 \sin^2\beta + |\mu|^2 \cos^2\beta - |A||\mu| \sin\beta \cos\beta y_t \cos(\phi_A + \phi_\mu)}} \right)$$

which assumes a simpler form for typical points in parameter space, as noted above (11). In keeping with prior notation, this defines

$$f(\phi_\mu, \phi_A) \equiv \frac{\sin\phi_\mu (|A| \sin\beta \cos\beta \cos\phi_A - |\mu| \cos^2\beta y_t \cos\phi_\mu)}{\sqrt{|A|^2 \sin^2\beta + |\mu|^2 \cos^2\beta - |A||\mu| \sin\beta \cos\beta y_t \cos(\phi_A + \phi_\mu)}}$$

^{||}for alternatives, see [39]

Table 2: Dependence of the asymmetry in $b \rightarrow s \gamma$ computed in the parton model and Fermi-Motion model on the minimum energy of the soft photon.

ξ	$(1 - \xi)E_{max} \text{ (GeV)}$	$ a_{parton} $	$ a_{fermi} $
1.00	0	0.008	0.008
0.30	1.85	0.010	0.010
0.15	2.24	0.011	0.012

Table 3: Definitions of the ξ -Dependent Coefficients. ‘(p)’ refers to the ‘parton model’ and ‘(f)’ includes ‘Fermi motion’.

ξ	$a_{27}^{(p)}$	$a_{87}^{(p)}$	$a_{28}^{(p)}$	$a_{27}^{(f)}$	$a_{87}^{(f)}$	$a_{28}^{(f)}$
1.00	1.06	-9.52	0.16	1.06	-9.52	0.16
0.30	1.17	-9.52	0.12	1.23	-9.52	0.10
0.15	1.31	-9.52	0.07	1.06	-9.52	0.04

3. The Operators $O_{2,7,8}$

We again follow [37] in calculating the coefficients $C_{7,8}$ of the operators $O_{7,8}$ from chargino loops:

$$C_{7,8\tilde{\chi}^\pm} = \frac{\alpha_w \sqrt{\alpha}}{2\sqrt{\pi}} \sum_{k=1}^6 \sum_{j=1}^2 \frac{1}{m_{\tilde{u}_k}^2} (G_{UL}^{jkb} - H_{UR}^{jkb}) (G_{UL}^{*jks} - H_{UR}^{*jks}) (F_{1,jk} + e_U F_{2,jk}) \\ - H_{UL}^{jkb} (G_{UL}^{*jks} - H_{UR}^{*jks}) \frac{m_{\tilde{\chi}_j^\pm}}{m_b} (F_{3,jk} + e_U F_{4,jk})$$

where $F_{1,2,3,4}$ are form factors and C_7 is obtained from the above by setting $\alpha = e^2/(4\pi)$ and $e_U = 2/3$; for C_8 $\alpha = g^2/(4\pi)$ and $e_U = 0$. Including the SM contributions given in [36], we obtain

$$\begin{aligned} C_2 &\approx 1.11 \\ C_7 &\approx -0.31 - 0.19 e^{i\phi_\mu} \\ C_8 &\approx -0.15 - 0.14 e^{i\phi_\mu} \end{aligned}$$

Note that C_2 , the real part of M_{12} , is not significantly affected by SUSY.

4. Soft Photons in $b \rightarrow s \gamma$

In 4.3.1 we noted that the energy dependence of the outgoing photon could have a significant effect on the BR and asymmetry. Here we quote the expression in [36] for the BR and asymmetry as functions of $C_{2,7,8}$ and ξ ($E_\gamma > (1 - \xi)E_{max}$):

$$\begin{aligned} BR(B \rightarrow X_s \gamma) &\approx 2.57 \times 10^{-3} K_{NLO}(\xi) \times BR(B \rightarrow X_c e \nu) / 10.5\% \\ \text{where} \\ K_{NLO}(\xi) &\equiv \sum_{i \leq j=2,7,8} (k_{ij}(\xi) Re(C_i C_j^*) + k_{77}^{(1)}(\xi) Re(C_7^{(1)} C_7^*)) \\ a_{b \rightarrow s \gamma} &= \frac{1}{|C_7|^2} (a_{27}(\xi) Im(C_2 C_7^*) + a_{87}(\xi) Im(C_8 C_7^*) + a_{28}(\xi) Im(C_2 C_8^*)) \end{aligned}$$

The ξ -dependent quantities are listed in Table 3 which we paraphrase from [36]; using this, it is straightforward to explicitly calculate the BR and asymmetry for the values of $C_{2,7,8}$ given above. The dependence is insignificant for a wide range of ξ and in two different models (see Table 2).

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